## Hypersonic Equations of Motion

| Grade | Time | Subject Area | Key Concepts |
| :--- | :--- | :--- | :--- |
| High School | 50 min | Physics | Speed/velocity <br> Equations of motion |
| Lesson Overview |  |  |  |

In this lesson, students will use physics equations of motion to analyze and compare two different hypersonic trajectories: the ballistic trajectory and the hypersonic glider trajectory. Students should have previously studied kinematics and projectile motion.

## NGSS Standards

HS-PS2-1 Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

## Learning Objectives

By the end of this lesson, students will be able to:

- Derive the equations of motions for both a ballistic trajectory and a hypersonic glider trajectory.
- Explain how the changing acceleration due to gravity during the flight path would affect their calculations.
- Compare and contrast the ballistic trajectory and the hypersonics glide trajectory.


## Essential/Overarching Question

How do ballistic flight paths differ from glide paths at hypersonic speeds?

## Key Vocabulary

Speed - the rate at which an object is moving. Speed is calculated by dividing the distance travelled by the time it took to travel that distance.

Speed of Sound - the rate at which sound moves through a medium. The speed of sound depends on both the density and the temperature of the medium. The speed of sound through air at $20^{\circ} \mathrm{C}\left(68^{\circ} \mathrm{F}\right)$ at sea level is $343 \mathrm{~m} / \mathrm{s}$ ( 767 mph ).

Mach - the ratio of the speed of an object to the speed of sound or how many times the speed of sound an object is moving. It is often followed by a number indicating the ratio; for example: Mach 1 is the speed of sound, Mach 2 is twice the speed of sound, Mach 5 is five times the speed of sound.

Sonic - speeds equal to the speed of sound (Mach 1).

Subsonic - speeds smaller than the speed of sound (less than Mach 1).

Transonic - speeds near (Mach 0.8-1.2) the speed of sound where drag is highest (e.g. sound barrier).

Supersonic - speeds greater than the speed of sound (Mach 1 and greater).

Hypersonic - speeds greater than five times the speed of sound (Mach 5 and greater).

Fluid - a substance with no fixed shape; a liquid, gas, or plasma. A substance that flows when an external force is applied to it.

Flow - the motion of a fluid (liquid, gas, or plasma) when it experiences unbalanced forces.

Trajectory - the curved path of an object after it is thrown or launched into the air.

Acceleration - the rate an object changes velocity; a change in velocity over time.

Acceleration due to Gravity - the acceleration, rate of change of velocity, at which an object free falls due to the gravitational attraction between the object and a celestial body; on Earth $\mathrm{g}=9.8 \mathrm{~m} / \mathrm{s}^{2}$ towards Earth's center.

Displacement - the change in position of an object.

Friction - resistance to movement. A force that opposes motion.

Height - an object's vertical position relative to a plane of reference ( 0 m ). Typically, the position above or below sea level or ground level.

Velocity - the speed of an object is a given direction. Speed is calculated by dividing the displacement of the object by the time interval in which the displacement occurred.

## Science Concepts Overview

Different objects move at different speeds. With such a large possible range of object speeds, it can sometimes be hard to compare them. In many cases, we compare the speed of an object to the speed of sound. The speed of sound tells you how fast a sound wave travels from its source to its receiver. The speed of sound depends on what medium the sound wave is traveling through (air, water, metal, etc.). It varies directly with both the density of the medium and temperature. The speed of sound of air at $20^{\circ} \mathrm{C}\left(68^{\circ} \mathrm{F}\right)$ at sea level is $343 \mathrm{~m} / \mathrm{s}$ ( 767 mph ).

When we compare the speed of an object to the speed of sound, we do so with a ratio called the Mach number. The Mach number is calculated by dividing the speed of an object by the speed of sound. The Mach number can be a whole number (Mach 3) or a decimal (Mach 0.6). Additionally, we categorize speeds by the size of their Mach number. Speeds less than Mach 1 are subsonic. Speeds greater than Mach 1 are supersonic. And speeds greater than Mach 5 are hypersonic.

There are two main flight paths of hypersonics projectiles, a ballistic trajectory and a hypersonics glider trajectory. An object following a ballistic trajectory is usually accelerated by a rocket booster and then follows a hyperbolic flight bath. An object following a ballistic trajectory is usually not maneuverable once it is launched. A hypersonic glider trajectory is initially launched by a rocket booster, but is not launched as high. It then uses engines to maneuver, or it may glide unpowered to its destination.

## Materials List

## Kinematics at Hypersonic Equations of Motion handout (one per student)

## Lesson Preparation

Prior to the lesson, the instructor should make copies of the Hypersonic Equations of Motionhandout and gather calculators if needed.

## Safety

There are no additional safety concerns beyond normal classroom procedures for this lesson.

## Procedure

## Engage (5 minutes)

1. For this lesson, students can work either individually or as a group.
2. Start the lesson by posing the following question:

- Previously, we studied projectile motion. What are different possible trajectory paths that we discussed?

3. Individually, or as a class, have the students read the Introduction on the Hypersonic Equations of Motion handout.
Explore (5 minutes)
4. Students will use the diagram to gather information about the two different trajectories and answer Analysis question 1 on the Hypersonic Equations of Motion handout:

- What do you notice about the trajectory? How would you describe the shape of the trajectory? Are their different phases of the motion (different shapes, different accelerations - engine powered vs. glide)?


## Explain (15 minutes)

5. Students will use their physics knowledge to further explain the difference between the ballistic and hyperbolic glide trajectories and answer Analysis questions 2-6 on the Hypersonic Equations of Motion handout:

- If you were to solve for how long it would take the vehicle to reach its destination, what information (variables) would you need?
- Using variables only, solve an equation for the time it would take for the vehicle to reach its destination.
- What assumptions are you making when using those equations?
- How do the two equations of motion compare?
- Acceleration due to gravity depends on the object's distance above Earth, decreasing as you move further away. For example, approximately: $\mathrm{g}=9.80$ $\mathrm{m} / \mathrm{s}^{2}$ at Earth's surface, $\mathrm{g}=9.50 \mathrm{~m} / \mathrm{s}^{2}$ at $100 \mathrm{~km}, \mathrm{~g}=8.68 \mathrm{~m} / \mathrm{s}^{2}$ at 400 km , and g $=6.59 \mathrm{~m} / \mathrm{s}^{2}$ at 1400 km . How would the changing acceleration due to gravity effect your calculations?


## Elaborate (5 minutes)

6. Students will use the analysis they did in questions 1-6 to compare the two trajectories and answer Analysis question 7 on the Hypersonic Equations of Motion handout:

- What are the pros and cons of the two different trajectories? Explain your reasoning.


## Evaluate (20 minutes)

7. Divide the students into small groups and assign each group to either the Ballistic Trajectory or the Hypersonic Glider Trajectory. Challenge the groups to give a 1minute sales pitch as to why their trajectory is the best. Groups will have 10 minutes to organize their sales pitch.

## STEM Career Connections

- Aerospace engineering
- Physicist
- Atmospheric scientist
- Testing engineer
- Pilots


## Extensions

To further explore the science and current events about hypersonics vehicles, students can read an article from Scientific American on developing ways to better detect glide paths. Hypersonic Weapons Can't Hide from New Eyes in Space - Scientific American

## References \& Resources

NASA Wallops. (2021, September 11). HOTShot sounding rocket launch [Video]. YouTube. https://www.youtube.com/watch?v=xGjdE7JdsK8

National Air and Space Museum. (n.d.) Hypersonic vehicles. https://howthingsfly.si.edu/structures-materials/hypersonic-vehicles
National Science Foundation. (2015, January 22). Projectile motion \& parabolas [Science of NFL football] [Video] YouTube. https://www.youtube.com/watch?v=HB4ws7RoA3M
Professor Dave Explains. (2017, February 2). Kinematics part 3: Projectile motion [Video]. YouTube. https://www.youtube.com/watch?v=aY8z2qO44WA

SciShow. (2019, July 8). A surprisingly simple secret to supersonic flight [Video]. YouTube. https://www.youtube.com/watch?v=kGefMLHJBKA
Sherman, J. (2022, January 18). Hypersonic weapons can't hide from new eyes in space. Scientific American. https://www.scientificamerican.com/article/hypersonic-weapons-cant-hide-from-new-eyes-in-space/
Talented Tuber. (2017, April 11). Difference between subsonic, supersonic and hypersonic speed [Video]. YouTube. https://www.youtube.com/watch?v=LBJ3tXCjzN0
TestTube 101. (2015, November 11). Flying at hypersonic speeds [Video]. YouTube https://www.youtube.com/watch?v=vL1qAfSOgic
The Economist. (2019, April 6). Gliding missiles that fly faster than Mach 5 are coming. The Economist. https://www.economist.com/science-and-technology/2019/04/06/gliding-missiles-that-fly-faster-than-mach-5-are-coming
UVA Engineering. (2021, September 27). "Need for speed: A hypersonics lecture"- Need for speed video contest [Video]. YouTube. https://www.youtube.com/watch?v=at2v5LJOSUU
Wright, D. \& Tracy, C. (2021, August 1). The physics and hype of hypersonic weapons. Scientific American. https://www.scientificamerican.com/article/the-physics-and-hype-of-hypersonic-weapons/

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## Hypersonic Equations of Motion

## 丸 Introduction

In many military missions, the goal is to move an object from one location to another without the opposition interfering. To run sneaky missions, you want to limit the time that your vehicle is detectable by the opposition. There are two ways to do so, move at high speeds (decreasing total travel time), and limit the distance that you are detectable (decreasing the time the opposition can see you). Depending on the way the vehicle is designed to travel, hypersonic vehicles can do both, travel at high speeds and decrease detectable time. Hypersonic means that the object is traveling at least five

Date:
 times the speed of sound ( $1715 \mathrm{~m} / \mathrm{s}=3836 \mathrm{mph}$ ). Hypersonics vehicles tend to travel using two different flight paths, a ballistic trajectory and a hypersonic glider trajectory as pictured above. A ballistic trajectory follows a projectile motion type path while a hypersonics glider does not. Use the information in the picture above to compare and contrast the two trajectories.
$\nrightarrow$ Analysis

## Ballistic Trajectory <br> Hypersonic Glider Trajectory

1. What do you notice about the trajectory? How would you describe the shape of the trajectory? Are their different phases of the motion (different shapes, different accelerations - engine powered vs. glide)?
2. If you were to solve for how long it would take the vehicle to reach its destination, what information (variables) would you need?
3. Using variables only, solve an equation for the time it would take for the vehicle to reach its destination.
4. What assumptions are you making when using those equations?
5. How do the two equations of motion compare?
6. Acceleration due to gravity depends on the object's distance above Earth, decreasing as you move further away. For example, approximately: $\mathrm{g}=9.80 \mathrm{~m} / \mathrm{s}^{2}$ at Earth's surface, $\mathrm{g}=9.50 \mathrm{~m} / \mathrm{s}^{2}$ at $100 \mathrm{~km}, \mathrm{~g}=8.68 \mathrm{~m} / \mathrm{s}^{2}$ at 400 km , and $\mathrm{g}=6.59$ $\mathrm{m} / \mathrm{s}^{2}$ at 1400 km . How would the changing acceleration due to gravity effect your calculations?
7. What are the pros and cons of the two different trajectories? Explain your reasoning.
